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LETTERS TO THE EDITOR

STUDIES ON MODEL UPDATING USING DIFFERENT FINITE ELEMENTS

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1. Introduction

A fairly accurate and representative computer model is required to predict the dynamic characteristics of the structure under study. The eigenvalue problems encountered in dynamics are of the type

$$[\mathbf{K}]\{\mathbf{u}\} = \omega^2[\mathbf{M}]\{\mathbf{u}\}, \tag{1}$$

where [**M**] and [**K**] are mass and stiffness matrices of size ($N \times N$), $\{\mathbf{u}\}$ is the eigenvector and ω is the corresponding natural frequency. Theoretically, equation (1) gives rise to N eigenpairs. Luckily, in all engineering problems the meaningful eigenpairs needed are very small in comparison with N. Significant discrepancies are often found when validating these models by comparing the numerical predictions with experimental values. These variations can be attributed to the analytical model because it is difficult to incorporate complex mechanical joints, accurate damping characteristics of the typical engineering structures into the finite element model. In such cases update strategies must be applied to obtain modified models. Model updating is concerned with globally tuning the elements of the spatial matrices in the light of the measured modal data. Preservation of the original banded characteristics in the updated matrices considerably reduces the computational requirements when solving eigenvalue problems of large size. The updated model will be applied more confidently to predict the internal stress levels and sensitivity to further design changes.

Baruch and Bar Itzhack [1] developed an optimal update for the stiffness matrix of a structure by using orthogonalized measured modes. Berman and Nagy [2] generated a model whose modes agree exactly with those used in identification, but improved analytical mass and stiffness matrices became dense and the elements were dramatically altered. Kabe [3] presented a stiffness matrix adjustment procedure that preserves the physical connectivity of the original model in the updated stiffness matrix. It is computationally involved because a large indefinite auxiliary linear system of equations must be solved. Smith and Beattie [4] considered quasi-Newton methods for stiffness updating which preserves the structural connectivity by overcoming the problems associated with the Kabe method.

In the present study, eigendata obtained from the finite element model is correlated with and corrected to the values of experimental modal analysis. Incomplete mode shape data obtained by conducting experimental modal analysis at 10 locations on a cantilever plate with holes is expanded to a full measured vector. To satisfy the orthogonality characteristic of true normal modes, the first four measured modes were made orthonormal with respect to the analytical mass matrix. The model updating procedure given by Smith and Beattie [4] is applied to improve the stiffness matrices of different problems with different types of elements.

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2. COMPUTATIONAL APPROACH

2.1. *Full mode computation*

The measured deflection data should be corrected to make them orthogonal with respect to the mass matrix because modal updating requires mass-orthonormalized modal vectors of size consistent with analytical model co-ordinates. The subsets of modal displacements for each mode can be related with analytical mass and stiffness matrices [6] to get the full measured vector and it can be represented as

$$\left\{\begin{bmatrix}[\mathbf{K}_{11}] & [\mathbf{K}_{21}]\\ [\mathbf{K}_{12}] & [\mathbf{K}_{22}]\end{bmatrix} - \omega_i^2\begin{bmatrix}[\mathbf{M}_{11}] & [\mathbf{M}_{21}]\\ [\mathbf{M}_{12}] & [\mathbf{M}_{22}]\end{bmatrix}\right\}\begin{Bmatrix}\{\boldsymbol{\phi}_{1i}\}\\ \{\boldsymbol{\phi}_{2i}\}\end{Bmatrix} = \{\mathbf{0}\}, \qquad (2)$$

where $\{\boldsymbol{\phi}_{1i}\}$ is the measured part of the eigenvector while $\{\boldsymbol{\phi}_{2i}\}$ is the unknown part; the lower matrices represent the unknown co-ordinates. Rearranging the lower matrix equation gives

$$\{\boldsymbol{\phi}_{2i}\} = -([\mathbf{K}_{22}] - \omega_i^2\,[\mathbf{M}_{22}])^{-1}([\mathbf{K}_{21}] - \omega_i^2\,[\mathbf{M}_{21}])\{\boldsymbol{\phi}_{i1}\}. \qquad (3)$$

The full measured modal vector can be obtained after rearranging the measured vector and expanded vector to the analytical model co-ordinates.

2.2. *Model updating procedure*

The modification procedure [4] minimizes changes to the analytical matrices within the constraints of the dynamic equations governing the vibration characteristics of the system and gives improved matrices which exactly predict the measured modal data. Using the spatial matrices $[\mathbf{K}_A]$ and $[\mathbf{M}_A]$ and modal data $[\boldsymbol{\omega}^2]$ and $[\boldsymbol{\phi}]$, generalization to a multiple-secant algorithm produces an optimal update identification method which minimizes

$$\sum_{i,j=1}^{N} \frac{(K_{ij} - K_{ij}^A)^2}{K_{ii}^A\,K_{jj}^A} \qquad (4)$$

subjected to $K\phi = M\phi\omega^2$, $K = K^{\mathrm{T}}$ and sparse $([\mathbf{K}])$ = sparse $([\mathbf{K}_A])$. The cost functional with a diagonal weighting matrix is

$$\|[\mathbf{D}]^{-1}(K - K_A)\,[\mathbf{D}]^{-1}\|_F^2,$$

where

$$[\mathbf{D}] = \mathrm{diag}\,(d_i) = \mathrm{diag}\,\sqrt{\mathbf{K}_{ii}^A}\,.$$

TABLE 1

Computational requirements

	Multiplications	Memory
(a) Eigenvalue problem	$Nb^2/2 + c[3Nbp + \frac{5}{2}Np^2 + \frac{1}{6}p^3] + Nbp$	$2Nb + 4Np$
(b) Mode expansion	$p[(N-n)^3/6 + 2N(N-n)]$	$2N^2 + 2(N-n)^2 + 2n(N-n)$
(c) Model updating	$Np[4N^2(p+2)] + 4N^2(p+1)$	$2N^2 + 2(N-n)^2 + 2n(N-n) + (N-n) + Np$

TABLE 2

Example results ((a), (b) and (c) refer to Table 1)

(a)	$0{\cdot}5 \times 10^6$	0·8 Mb
(b)	$6{\cdot}5 \times 10^6$	3·0 Mb
(c)	1000×10^6	2·0 Mb

$N = 220$, $n = 10$, $b = 60$, $p = 4$, $c = 8$ (no. of iterations with a PC 486 machine)

Lagrangian multipliers incorporate the constraints into an above extended cost function. Minimization of the Lagrangian function produces a system of linear equations to solve for the Lagrange multipliers. The final update equation can be obtained by adding the error matrix elements, obtained by using the Lagrangian constants, to the original stiffness matrix elements and it is given by

$$K_{ij} = K_{ij}^A + d_i\, d_j\, \{[\mathbf{P}]_i\,[\mathbf{D}]\,[\boldsymbol{\phi}]_i\,\{\boldsymbol{\gamma}_i\}_j + [[\mathbf{P}]_j\,[\mathbf{D}]\,[\boldsymbol{\phi}]_j\,\{\boldsymbol{\gamma}_j\}]_i\}, \qquad \text{for } i, j = 1, \ldots, N, \tag{5}$$

where $[\boldsymbol{\phi}]$ = measured modal matrix, $[\boldsymbol{\gamma}]$ = matrix of Lagrangian constants, $[\mathbf{P}]_i$ = diagonal matrix of ones and zeros which masks a vector with the scarcity pattern of the ith row of $[\mathbf{K}_A]$.

An iterative conjugate gradient method was used to solve the auxiliary problem (Np, Np). A reduced storage solution can be used by taking advantage of repetitive substructure patterns without storing the coefficient matrix explicitly. A maximum of Np iterations are necessary for better convergence of the solution.

2.3. *Computational requirements*

Computational requirements are listed in Table 1, numerical examples are given in Table 2.

3. APPLICATIONS

3.1. *Rectangular plate with holes using a 3-D plate element*

A plate of size 300 mm × 150 mm × 3 mm with circular and rectangular holes was considered to allow application of the updating procedure. It was descretized (Figure 1) with 42 nodes and 54 elements using triangular 3-D plate elements with 6 degrees of freedom per node. A finite element model of 222 active degrees of freedom with a maximum band width of 60 was used to get the first four natural frequencies using a

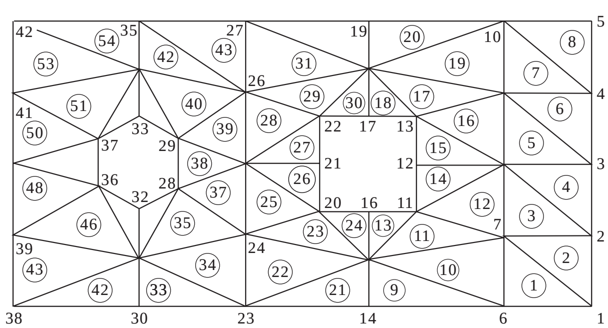

Figure 1. Finite element discretization of plate.

TABLE 3

Comparison of natural frequencies of the plate (triangular plate element)

Mode	FEM (Hz)	Experimental (Hz)	Improved FEM (Hz)
1	25·02	23·72	23·68
2	109·32	101·15	101·14
3	157·15	151·45	150·92
4	358·32	334·89	333·98

simultaneous iteration [7] scheme and the values are shown in Table 3. The first four eigenvalues converged with 8 iterations.

3.1.1. *Modal test*. The testing arrangement including test specimen and instrumentation used for modal analysis is as given in Figure 2. Frequency response functions (FRF, acceleration/force) were captured at 12 locations on the boundary of the plate, keeping the accelerometer fixed at the free end and hitting at all the points using an impact hammer. This resulted in one point FRF and 11 transfer FRFs. By using a multi degree of freedom (MDOF) curve fit, based on a complex exponential least squares algorithm, modal parameters were estimated from the driving FRF. The first four natural frequencies are tabulated in Table 3. The full measured mode vector (222×1) was calculated using equation (3) from the incomplete measured mode vector (10×1) for the first four frequencies. The expanded modal vectors and eigenvectors obtained by eigenvalue analysis were comparable.

3.1.2. *Improved solution*. The modified stiffness matrix is calculated using equation (5). This improved stiffness matrix, having the same characteristic of the original stiffness matrix (222×60), along with original mass matrix was used to obtain improved eigenvalues using the simultaneous iteration scheme. The improved frequencies are listed in Table 3.

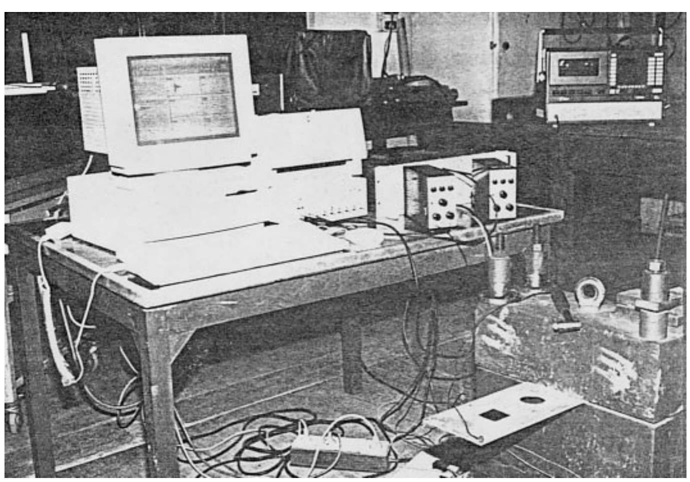

Figure 2. Experimental set-up (rigid clamped–free boundaries).

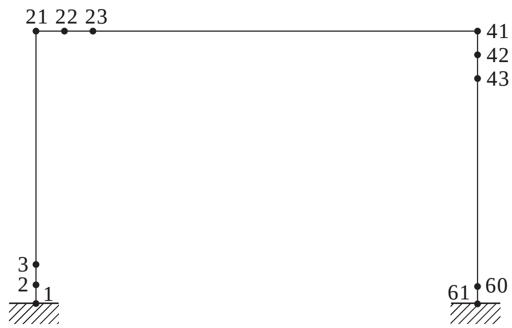

Figure 3. Finite element discretization of EOT crane.

3.2. *EOT crane using 3-D beam elements*

A one meter height EOT crane was analyzed with 3-D beam elements with 6 degree of freedom (DOF)/node to test the applicability of the stiffness matrix modification procedure. The structure was discretized with 61 nodes and 60 elements (Figure 3). The active degrees of freedom were 354 and the maximum band width was 12. The first 10 eigenvalues were obtained with 15 iterations and are listed in Table 4. A 10% variation was assumed for the first four natural frequencies to get the corresponding experimental values. The improved stiffness matrix elements using the first four modes were obtained using equation (5). The improved natural frequencies are given in Table 4 for comparison. The identified four modes are in good agreement with the assumed measured modes. The other frequencies were also slightly changed with the modification process.

3.3. *Rectangular plate with brick element*

A cantilever mild steel plate of size 200 mm × 100 mm × 6 mm was modelled with 90 nodes and 32 elements. A 3-D brick element with 3 DOF/node was used for discretization. An eigenvalue problem of size 240 × 42 is solved for 10 eigenvalues and orthonormalised eigenvectors. A 5% variation in the first four natural frequencies was assumed to approximate the measured values. These four eigenvalues and vectors were used along with analytical stiffness and mass matrices to improve the finite element model using equation (5). The eigensolution with improved stiffness matrix and original mass matrix produced

TABLE 4

Comparison of natural frequencies (Hz) for EOT crane (beam element)

Mode no.	FEM	Expected values	FEM improved
1	19·612	21·574	21·5051
2	31·353	34·488	34·5589
3	38·504	42·354	41·5073
4	56·240	61·864	61·8264
5	74·711	–	77·9431
6	157·167	–	157·89
7	169·86	–	171·956
8	239·148	–	238·878
9	239·676	–	240·4152
10	285·752	–	284·861

TABLE 5

Comparison of natural frequencies (Hz) of plate (brick element)

Mode no.	FEM	Expected	FEM improved
1	318·785	350·663	352·53
2	1232·883	1356·171	1356·355
3	1810·623	1991·685	1991·692
4	1918·020	2109·822	2109·775
5	3901·184	–	3983·010
6	5127·128	–	5267·262
7	6549·826	–	6554·411
8	6693·076	–	6746·727
9	6791·341	–	6819·220
10	7128·748	–	7245·297

the first four natural frequencies which are in better agreement with the expected measured values, as given in Table 5.

4. CONCLUSIONS

Analytical stiffness matrices of different problems are modified in the light of incomplete modal data to reproduce the identified measured frequencies using different types of finite elements: a triangular plate element with 6 DOF/node, a 3-D beam element and a brick element. The original characteristics of the stiffness matrix are retained after modification. Unidentified modes are also changed to some extent due to stiffness modification.

Incomplete modal vectors are expanded to full vectors, consistent with the finite element degrees of freedom. Self-compatibility of these measured data is checked to satisfy the orthogonal property of the true normal modes. The off diagonal elements showed an error of 9% in the fourth mode. However this procedure will lead to convergence problems if the measured data contains errors of substantial magnitude.

REFERENCES

1. M. BARUCH and I. Y. BAR ITZHACK 1978 *American Institute of Aeronautics and Astronautics* **16**, 346–351. Optimal weighted orthogonolization of measured modes.
2. A. BERMAN and E. J. NAGY 1983 *American Institute of Aeronautics and Astronautics* **21**, 1168–1173. Improvement of a large analytical model using test data.
3. A. M. KABE 1985 *American Institute of Aeronautics and Astronautics* **23**, 1431–1436. Stiffness matrix adjustment using mode data.
4. S. W. SMITH and C. A. BEATTIE 1991 *American Institute of Aeronautics and Astronautics* **29**, 119–126. Secant-method adjustment for structural models.
5. D. J. EWINS 1988 *Modal Testing: theory and practice*. Research Studies Press.
6. R. J. GUYAN 1964 *American Institute of Aeronautics and Astronautics* **3**, 380. Reduction of stiffness and mass matrices.
7. V. RAMAMURTI 1996 *Computer Aided Mechanical Design and Analysis*. New Delhi: Tata McGraw Hill; third edition.